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A five-dimensional toy-model for light hadron excitations

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A typical holographic model of QCD (Erlich et al., PRL 95, 261602 (2005))

$$S = \int d^5x \sqrt{g}\, \mathrm{Tr}\left\{ |DX|^2 + 3|X|^2 - \frac{1}{4g_5^2}(F_L^2 + F_R^2) \right\}$$

$$D_\mu X = \partial_\mu X - iA_{L\mu}X + iXA_{R\mu},\ A_{L,R} = A^a_{L,R}t^a,\ F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu].$$

For $N_f = 2$ $t^a = \sigma^a/2$

Hard wall model: AdS_5 space with the metric $\boxed{ds^2 = \frac{R^2}{z^2}(-dz^2 + dx^\mu dx_\mu)}$

where R is the AdS curvature radius, cut at z coordinate: $0 < z \le z_m$

The fifth coordinate corresponds to the energy scale: $Q \sim 1/z$

Because of the conformal isometry of the AdS space, the running of the QCD gauge coupling is neglected until an infrared scale $Q_m \sim 1/z_m$. At $z = z_m$ one imposes certain gauge invariant boundary conditions on the fields.

Equation of motion for the scalar field $X \sim \bar{q}q$

$$\frac{1}{z^5}3X = \frac{1}{z^3}\partial_\mu\partial^\mu X - \partial_z \frac{1}{z^3}\partial_z X$$

Solution independent of usual 4 space-time coordinates

$$X_0(z) = \frac{1}{2}Mz + \frac{1}{2}\Sigma z^3$$

where ***M*** is identified with the quark mass matrix and ***Σ*** with the condensate.

Denoting

$$X_0(z) = \frac{1}{2}v(z)\mathbf{1}, \qquad v(z) = mz + \sigma z^3$$

the equations of motion for the vector fields are (in the axial gauge)

$$\left[\partial_z\left(\frac{1}{z}\partial_z V_\mu^a(q,z)\right) + \frac{q^2}{z}V_\mu^a(q,z)\right]_\perp = 0$$

where $V(q,z) = \int d^4x \;\; e^{iqx}V(x,z)$

$$\left[\partial_z\left(\frac{1}{z}\partial_z A_\mu^a\right) + \frac{q^2}{z}A_\mu^a - \frac{g_5^2 v^2}{z^3}A_\mu^a\right]_\perp = 0$$

Due to chiral symmetry breaking

They have normalizable solutions only for discrete values of 4d momentum $q^2 = m_n^2$

A common bfeature: Spectrum appears due to nontrivial 5D background;
there is infinite number of states

Vacuum sector

Introduce 5D scalar field $\varphi \sim G_{\mu\nu}^2$

Trace anomaly: $$4\varepsilon_{\text{vac}} = \left\langle \Theta_\mu^\mu \right\rangle_{\text{n.p.}} = \frac{\beta(\alpha_s)}{4\alpha_s} \left\langle G_{\mu\nu}^2 \right\rangle_{\text{n.p.}} + \ldots$$

Let us write the following effective model (i.e. valid below some energy scale $1/z_0$)

$$S_{\text{vac}} = \int d^4x dz \left(\frac{1}{2} \partial_A \varphi \partial^A \varphi + \frac{1}{2} m^2 \varphi^2 - \frac{1}{4} \lambda \varphi^4 \right)$$

$$\eta_{AB} = (1, -1, -1, -1, -1), \quad A = 0, 1, 2, 3, 4$$

Making use of the scaling

$$x \to \frac{x}{m}, \qquad \varphi \to \frac{m}{\sqrt{\lambda}} \varphi,$$

the action can be rewritten as

$$S_{\text{vac}} = \frac{1}{\lambda m} \int d^4x dz \left(\frac{1}{2} \partial_A \varphi \partial^A \varphi + \frac{1}{2} \varphi^2 - \frac{1}{4} \varphi^4 \right)$$

By assumption the selfinteraction is weak $\quad \lambda m \ll 1$

The classical equation of motion is

$$\partial_\mu^2 \varphi - \partial_z^2 \varphi - \varphi(1 - \varphi^2) = 0$$

We assume that the vacuum solution does not depend on the usual space-time coordinates,

$$\varphi(x_\mu, z) = \varphi(z)$$

The equation above has then a kink solution

$$\varphi_{\rm kink} = \pm \tanh(z/\sqrt{2})$$

Translational invariance along the *z*-direction is broken!

Thus, different energy scales are now not equivalent. The effect is essential at large *z* (small energies) but disappears at small *z* (high energies)

Consider the particle-like excitations by varying $\varphi = \varphi + \varepsilon$

Assuming $\varepsilon(x_\mu, z) = e^{ipx}\varepsilon(z)$ with $p^2 = M^2$ and retaining only linear part,

$$\left(-\partial_z^2 + 3\tanh^2(z/\sqrt{2}) - 1\right)\varepsilon_n = M_n^2\varepsilon_n$$

There are two normalizable discrete states,

$$\varepsilon_0 = \frac{1}{\cosh^2(z/\sqrt{2})}, \qquad M_0^2 = 0;$$

$$\varepsilon_1 = \frac{\tanh(z/\sqrt{2})}{\cosh(z/\sqrt{2})}, \qquad M_1^2 = \frac{3}{2}.$$

Continuum begins at $p^2 = 2$

"glueball"?

This suggests a natural limitation – the model is valid below the scale of the second scalar glueball.

Coupling to bosons

For simplicity, we consider the scalar case only

$$S_{\rm bos} = \int d^4x dz \left(\frac{1}{2}\partial_A \Phi \partial^A \Phi - \frac{G}{2}\varphi^2 \Phi^2 \right)$$

Making the rescaling above and $\Phi \longrightarrow m^{3/2}\Phi$ the corresponding Lagrangian reads

$$\mathcal{L}_{\rm bos} = \frac{1}{2}\left(\partial_A \Phi \partial^A \Phi - \frac{G}{\lambda}\varphi^2 \Phi^2 \right)$$

Consider the particle-like excitations $\Phi(x_\mu, z) = e^{ipx} f(z), \, p^2 = M^2$

$$\left(-\partial_z^2 + \frac{G}{\lambda}\tanh^2(z/\sqrt{2}) \right) f_n = M_n^2 f_n$$

The discrete spectrum is

$$M_n^2 = \frac{1}{2}\left[\sqrt{1+\frac{8G}{\lambda}}\left(n+\frac{1}{2}\right)-\left(n+\frac{1}{2}\right)^2-\frac{1}{4}\right]$$

$$f_n = \cosh^{n-s}(z/\sqrt{2})\times$$
$$F\left[-n, 2s+1-n, s+1-n, \frac{1-\tanh(z/\sqrt{2})}{2}\right]$$

where *F* is hypergeometric function and

$$s = \frac{1}{2}\left(\sqrt{1+\frac{8G}{\lambda}}-1\right),$$

$$n = 0, 1, 2, \ldots, \qquad n < s.$$

The continuum sets in at $n = s$

At $G/\lambda \gg 1$ the spectrum is Regge like.

There is a phenomenological selfconsistency: The end of the discrete light meson spectrum and the expected scale of the second scalar glueball (about 2.5 GeV) approximately coincide.

Coupling to fermions

$$S_{\text{ferm}} = \int d^4x dz \left(i\bar{\Psi}\Gamma^A \partial_A \Psi - h\varphi\bar{\Psi}\Psi \right)$$

Here $\Gamma^\mu = \gamma^\mu,\ \Gamma^4 = -i\gamma^5$ After our rescaling and $\psi \to m^2\psi$

$$\mathcal{L}_{\text{ferm}} = i\bar{\Psi}\Gamma^A \partial_A \Psi - \frac{h}{\sqrt{\lambda}}\varphi\bar{\Psi}\Psi$$

Let us find particle-like excitations $\Psi_{L,R}(x_\mu, z) = e^{ipx} U_{L,R}(z)$ for the left and right components $\gamma_5 \Psi_{L,R} = \pm\Psi_{L,R}$

$$\left(\pm\partial_z + \frac{h}{\sqrt{\lambda}} \tanh(z/\sqrt{2}) \right) U_{L,R} = M U_{L,R}$$

The equation is known to possess a normalizable zero-mode solution

$$M = 0, \qquad U_L = \cosh^{-\frac{\sqrt{2}\,h}{\sqrt{\lambda}}}(z/\sqrt{2}), \qquad U_R = 0$$

This mode is located near $z = 0$ There is also an asymptotic solution

$$z \to \infty : \qquad M = \frac{h}{\sqrt{\lambda}}, \qquad U_{L,R} = C_{L,R}$$

Conclusion

Five-dimensional approach can be used for construction of effective models of QCD describing the trace anomaly, scalar glueball, and emergence of Regge like meson spectrum with finite number of states.